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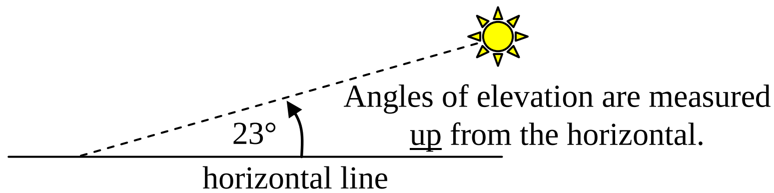
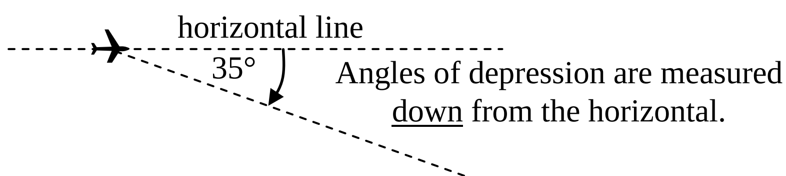
Applications of Right Triangle Trigonometry: Angles of Elevation and Depression

Preliminary Information: On most maps, it is customary to orient oneself relative to the direction north: for this reason, north is almost always indicated on every map.   Likewise, when working with real-life trigonometry problems, it is very common to orient angles relative to a horizontal line.

An angle of elevation refers to the acute angle a line (or ray, segment, etc.) makes with a horizontal line, when measured above the horizontal (hence an angle of *elevation*). For example, the sun's rays could form a 23° angle of elevation (above the horizon).

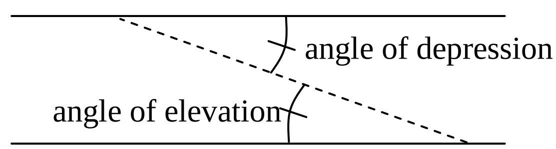


An angle of depression refers to the acute angle a line makes with a horizontal line, when measured below the horizontal (hence an angle of *depression*). For example, an airplane pilot could look down and see a feature on the ground below at a 35° angle of depression (below the horizon).



Angles of elevation and depression typically have their vertex at the point where an observer is positioned. In the previous example, notice that the vertex of the 35° angle is located at the pilot's location.

Because horizontal lines are everywhere parallel, angles of depression and elevation are numerically equivalent because they form alternate interior angles of parallel lines:

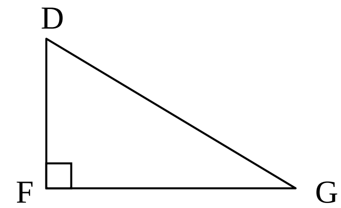


Students should always be encouraged to consider the following two ideas when they see either phrase mentioned in a problem:

- You may always draw an additional horizontal line on any diagram extending from any point in the diagram. Just as you did in Geometry, drawing such an *auxiliary line* can help to make a complex problem simpler.

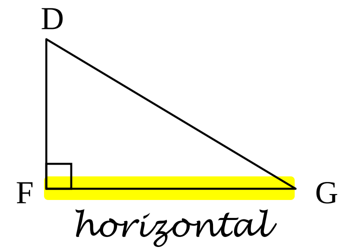
- The most common error students make when they encounter these terms is they mark an angle relative to a vertical line (such as an angle with a wall, building, or tree) instead of with a horizontal line. As stated earlier, always feel free to draw in an auxiliary horizontal line.

Part I) Model Problems



Example 1: Consider right ΔDFG pictured at right. Classify each angle as an angle of elevation, an angle of depression, or neither.

Step 1: Highlight the horizontal line(s) in the figure.



Step 2: Determine which acute angles are formed with the <u>horizontal</u>. In this example, $\angle F$ and $\angle G$ are formed with the horizontal, but only $\angle G$ is acute.

Step 3: Classify each angle:

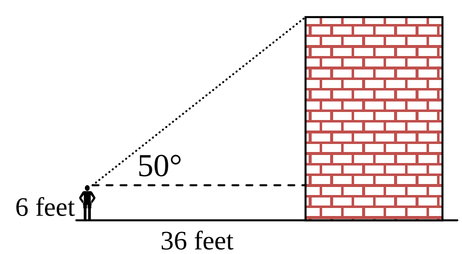
- $\angle D$ is <u>neither</u> an angle of depression nor an angle of elevation, as it is formed with <u>vertical</u> line segment $\overline{DF}$.

- $\angle F$ is formed with a horizontal line segment, but it is not an acute angle. So it is <u>neither</u>. (In reality, there would be no harm in specifying it as a 90° angle of elevation, but it is simpler just to say that $\angle F$ is a right angle.

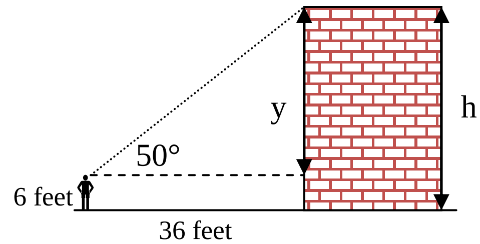
- $\angle G$ is the only acute angle measured from the horizontal; because line segment $\overline{DG}$ is above the horizontal, it is an <u>angle of elevation</u>.

Example 2: Michael, whose eyes are six feet off the ground, is standing 36 feet away from the base of a building, and he looks up at a 50° angle of elevation to a point on the edge of building's roof. To the nearest foot, how tall is the building?

Step 1: Make a detailed sketch of the situation. Make sure to include auxiliary horizontal lines as needed.



Step 2: Assign variables to represent the relevant unknowns. In this example, we shall use the variable y to represent the vertical leg of the right triangle, and h to represent the height of the building.



Step 3: Use SOHCAHTOA to solve for the unknown side of the right triangle:

$$\tan 50° = \frac{y}{36}$$

$$y = 36\tan 50°$$ (Note that units of feet were dropped for simplicity.)

$$y = 36(1.19175)$$

$$y = 42.90\ ft$$

Step 4: Determine the height of the building: Since Michael's eyes are six feet from the ground, we must add six feet to variable y to get h:

$$h = 6 + y$$

$$h = 6 + 42.90$$

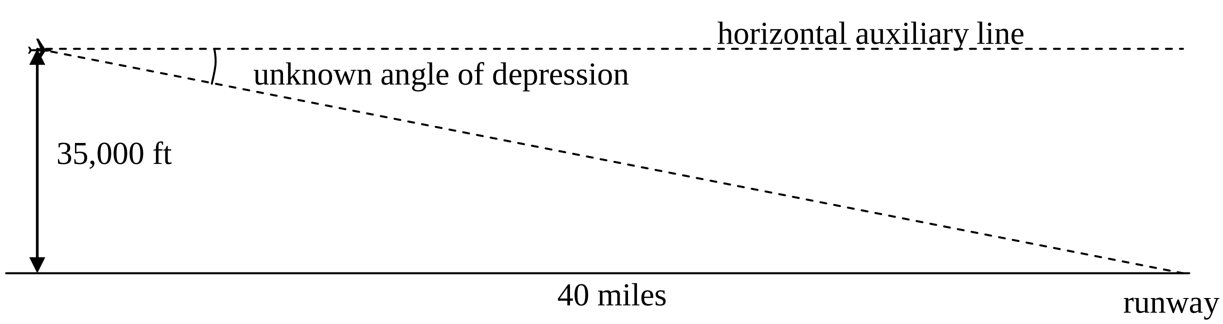
$$h = 48.90\ ft$$

$$h = 49\ ft \quad (rounded)$$

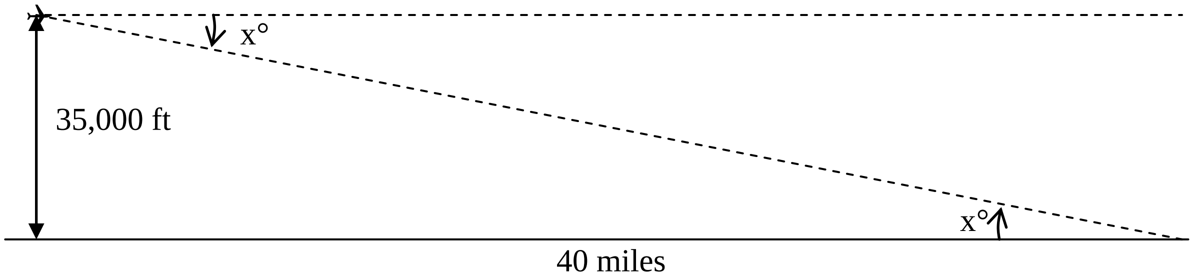
Step 5: Check for reasonableness: If Michael were looking up at a 45° angle of elevation, y would be 36 feet due to the isosceles triangle created. Because he is looking up at a greater angle, it is reasonable that y is greater than 36 feet. Adding 6 feet accounts for the fact that his eyes are 6 feet from the ground.

Example 3: A pilot is traveling at a height of 35,000 feet above level ground. According to her GPS, she is 40 miles away from the airport runway, as measured along the ground. At what angle of depression will she need to look down to spot the runway ahead?

Step 1: Make a detailed sketch of the situation. Make sure to include auxiliary horizontal lines as needed.



Step 2: Assign variables to represent the relevant unknowns. In this example, we shall use the variable x to represent the unknown angle of depression:



Because the ground is horizontal, and the auxiliary line is horizontal, we can properly assume that both angles marked x in the figure are congruent, as they are both alternate interior angles of parallel lines.

Step 3: Use SOHCAHTOA to solve for the unknown angle:

$$\tan x° = \frac{35{,}000\, ft}{40 miles}$$

Because we have mixed units, we recall that there are 5280 feet in a mile to convert 40 miles to feet:

$$\tan x° = \frac{35{,}000\, ft}{40 miles \cdot \frac{5280 ft}{mile}} = \frac{35{,}000\, ft}{211{,}200\, ft} = \frac{35{,}000}{211{,}200}$$

To solve for the unknown angle, we use the inverse tangent function:

$$\tan x° = \frac{35{,}000}{211{,}200}$$

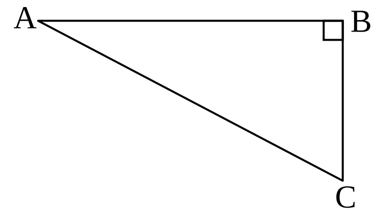
$$x° = \tan^{-1}\left(\frac{35{,}000}{211{,}200}\right)$$

$$x° = 9.41°$$

Step 4: Check for reasonableness: 40 miles is much larger than 35,000 feet, so it seems reasonable that the pilot would look down at an angle of only a few degrees.

Part II) Practice Problems

1. Classify each of the three angles in the figure at right as an angle of elevation, an angle of depression, or neither.



2. Multiple-Choice: A 15 foot ladder rests against a tree on level ground and forms a 75° angle of elevation. Where is the correct location of the 75° angle?

A) Between the ladder and the ground
B) Between the ladder and the tree
C) Between the tree and the ground
D) It is not possible to place a 75° angle on such a figure.

3. Tammi Jo, whose eyes are five feet off the ground, is standing 50 feet away from the base of a building, and she looks up at a 73° angle of elevation to a point on the edge of building's roof. To the nearest foot, how tall is the building?

4. A pilot is traveling at a height of 30,000 feet above level ground. She looks down at an angle of depression of 6° and spots the runway. As measured along the ground, how many miles away is she from the runway? Round to the nearest tenth of a mile.

5. A dog, who is 8 meters from the base of a tree, spots a squirrel in the tree at an angle of elevation of 40°. What is the direct-line distance between the dog and the squirrel?

6. A ship is on the surface of the water, and its radar detects a submarine at a distance of 238 feet, at an angle of depression of 23°. How deep underwater is the submarine?

7. The sun is at an angle of elevation of 58°. A tree casts a shadow 20 meters long on the ground. How tall is the tree?

8. Two observers on the ground are looking up at the top of the same tree from two different points on the horizontal ground. The first observer, who is 83 feet away from the base of the tree, looks up at an angle of elevation of 58°. The second observer is standing only 46 feet from the base of the tree. (Note: you may ignore the heights of the observers and assume their measurements are made directly from the ground.)

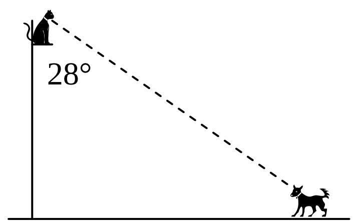
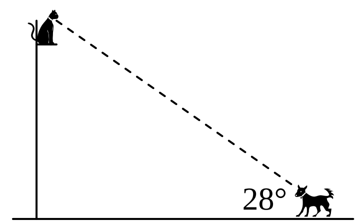
a) How tall is the tree, to the nearest foot?

b) At what angle of elevation must the second observer look up to see the top of the tree?

9. Error Analysis: Consider the following problem, which Stephanie and Adam are both trying to solve:

> "A cat, who has climbed a tree, looks down at a dog at a 28° angle of depression. If the dog is 34 meters from the base of the tree, how high up is the cat?"

The first steps of their work are shown below. Analyze their work and determine who, if anyone, has set it up correctly.

Stephanie's work	Adam's work
 34 meters $\tan 28° = \frac{34}{x}$	 34 meters $\tan 28° = \frac{x}{34}$

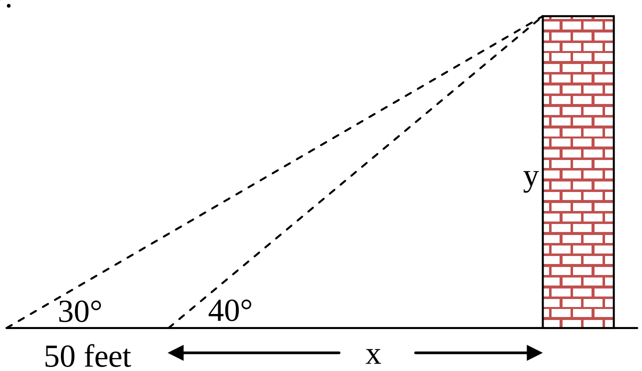
10. Complete problem 9: How high up in the tree is the cat?

Part III) Challenge Problems

11. A person starts out 17 miles from the base of a tall mountain, and looks up at a 4° angle of elevation to the top of the mountain. When they move 12 miles closer to the base of the mountain, what will be their angle of elevation when they look to the top? Answer to the nearest degree.

12. A pilot maintains an altitude of 25,000 feet over level ground. The pilot observes a crater on the ground at an angle of depression of 5°. If the plane continues for 16 more miles, what will be the angle of depression to the crater? Answer to the nearest degree.

13. An observer on the ground looks up to the top of a building at an angle of elevation of 30°. After moving 50 feet closer, the angle of elevation is now 40°. Consider the diagram below:



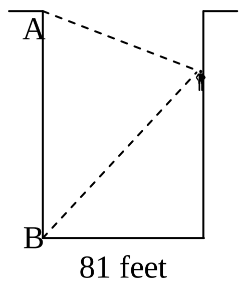
a) Set up an equation representing the situation from the first vantage point. Your equation will incorporate the 30° angle, x, y, and the 50 feet.

b) Set up an equation representing the situation from the second vantage point. Your equation will incorporate the 40°, x, and y.

c) You now have two equations in two variables. Solve them simultaneously to determine the value of x, the distance from the second vantage point to the base of the building.

d) Solve for y, the height of the building.

14. Two observers (located at points A and B in the diagram) are watching a climber on the opposite face of a chasm. The chasm is 81 feet wide. When observer A looks down to the bottom of the opposite wall of the chasm, he must look down at an angle of depression of 51°. However, observer A sees the climber at an angle of depression of 20°. Observer B will see the climber at what angle of elevation?



Part IV) Answer Key

1. Angle A is an angle of depression; B is neither; C is neither.
2. A
3. 169 feet
4. 54.1 miles
5. 10.4 meters
6. 93 feet
7. 32 feet
8. a) 133 feet

   b) 71°
9. Stephanie's work is incorrect because she drew the 28° angle relative to the vertical tree. Adam's work is correct because he drew the 28° angle relative to the horizontal. Technically, Adam drew an angle of <u>elevation</u> of 28°, but because all horizontal lines are parallel, his placement of the angle and the equation he wrote will, in fact, yield the correct solution.
10. The cat has climbed up approximately 18 meters.
11. 13°
12. 7°
13.

a) $\tan 30° = \dfrac{y}{50 + x}$	b) $\tan 40° = \dfrac{y}{x}$
c) $y = 50\tan 30° + x \cdot \tan 30°$ $y = x \cdot \tan 40°$ *Therefore* $x \cdot \tan 40° = 50\tan 30° + x \cdot \tan 30°$ $x \cdot \tan 40° - x \cdot \tan 30° = 50\tan 30°$ $x(\tan 40° - \tan 30°) = 50\tan 30°$ $x = \dfrac{50\tan 30°}{\tan 40° - \tan 30°}$ $x = 110.3\,ft$	d) $y = x \cdot \tan 40°$ $y = 110.3\tan 40°$ $y = 92.5\,ft$

14. 41°