www.MathWorksheetsGo.com
On Twitter: twitter.com/engagingmath
On FaceBook: www.mathworksheetsgo.com/facebook

Web Resources

Using the inverse sine, cosine, and tangent to find an angle
Right Triangle Calculator
Inverse Sine Calculator
Inverse Cosine Calculator
Inverse Tangent Calculator
Sine Cosine Tangent Chart

© www.MathWorksheetsGo.com All Rights Reserved
Commercial Use Prohibited

Terms of Use: By downloading this file you are agreeing to the Terms of Use Described at http://www.mathworksheetsgo.com/downloads/terms-of-use.php .

Graph Paper Maker (free): www.mathworksheetsgo.com/paper/

Online Graphing Calculator(free): www.mathworksheetsgo.com/calculator/

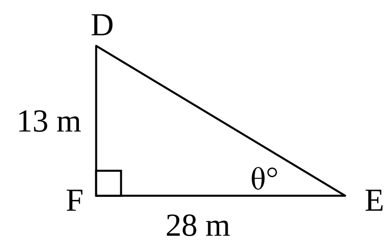
Right Triangles and SOHCAHTOA: Finding the Measure of an Angle Given any Two Sides

Preliminary Information: "SOH CAH TOA" is an acronym to represent the following three trigonometric ratios or formulas:

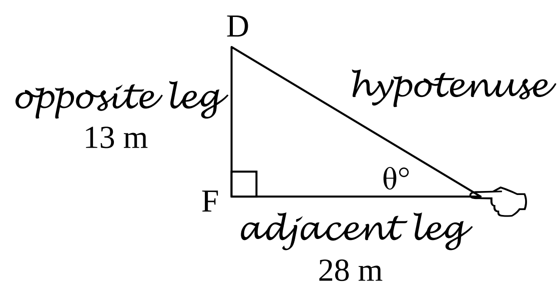
$$\sin\theta = \frac{opposite\ leg}{hypotenuse}\ ;\ \cos\theta = \frac{adjacent\ leg}{hypotenuse}\ ; \tan\theta = \frac{opposite\ leg}{adjacent\ leg}$$

Part I) Model Problems

Example 1: Consider right ΔDFE pictured at right. We know two sides, and our goal is to determine the measure of the unknown angle θ.



Step 1: Place your finger on the unknown angle, and then label the three sides: the hypotenuse is always the longest side; the side you are not touching is the opposite leg; and the remaining side you are touching is the adjacent leg.



Step 2: We need to determine which trigonometric ratio to use: the sine, the cosine, or tangent. It is recommended that you write "SOH CAH TOA" on your paper:

SOH CAH TOA

Step 3: Ask yourself, "Which sides do I know?" In this example, we know that the Adjacent leg is 28 m, and we know the Opposite leg is 13 m. To indicate that we know the Adjacent leg, underline both A's, and to indicate that we know the Opposite leg, underline both O's:

S<u>O</u>H C<u>A</u>H T<u>OA</u>

Step 4: Consider which of the three ratios has the most information: we have one piece of information for the sine (one underline), only one piece of information for the cosine (one underline), yet we have two pieces of information for the tangent (two underlines). We are therefore going to use the tangent ratio formula:

$$\tan\theta = \frac{\textit{opposite leg}}{\textit{adjacent leg}}$$

Step 5: Substitute the known information into the formula:

$$\tan\theta = \frac{\textit{opposite leg}}{\textit{adjacent leg}} \Rightarrow \tan\theta = \frac{13m}{28m} \Rightarrow \tan\theta = \frac{13}{28}$$

Step 6: Determine the angle θ that satisfies this equation. There are generally two methods for finding this unknown angle:

Method 1: Table Lookup (approximate to the nearest degree)

We start with the equation

$$\tan\theta = \frac{13}{28}$$

First, we can approximate the fraction with a decimal:

$$\tan\theta = \frac{13}{28} = 0.4643$$

Next, we can examine a table of values from a chart and look for the closest "match" in the tangent column:

Angle	Sine	Cosine	Tangent
24°	0.40674	0.91355	0.44523
25°	0.42262	0.90631	0.46631
26°	0.43837	0.89879	0.48773

In which case, we pick an angle of 25°:

Angle	Sine	Cosine	Tangent
24°	0.40674	0.91355	0.44523
25°	0.42262	0.90631	**0.46631**
26°	0.43837	0.89879	0.48773

So we conclude that θ = 25° to the nearest degree.

Method 2: Inverse Function on a Calculator (more accurate)

We rewrite the equation using the inverse tangent as

$$\theta = \tan^{-1}\left(\tfrac{13}{28}\right)$$

which is pronounced "theta is the inverse tangent of thirteen twenty-eighths."

What the inverse tangent function does is tell us what angle has a tangent of 13/28.

To enter this on a calculator, you typically type the SHIFT , INV or 2^{nd} key, and then the tan or $\tan^{-1}$ key (make sure your calculator is in degrees mode):

$$\theta = \tan^{-1}\left(\tfrac{13}{28}\right) = 24.9048°$$

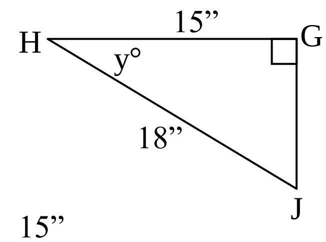
You could also use an online inverse tangent calculator

Step 7: Check for reasonableness: since the 13 m side is the shortest side, it makes sense that the opposite angle would be smallest. This answer seems reasonable. Notice that both methods resulted in approximately the same angle.

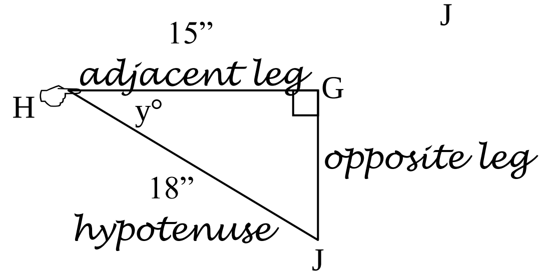
Note: it is also possible to directly check your answer in the equation:

$$\tan 24.9048° = \frac{13}{28} \qquad ☺$$
$$0.4643 = 0.4643$$

Example 2: Consider right ΔGHJ pictured at right. We know two sides, and our goal is to determine the degree measure of the unknown angle **y**.



Step 1: Place your finger on the unknown angle y, and then label the three sides: the <u>hypotenuse</u> is always the longest side; the side you are <u>not</u> touching is the <u>opposite leg</u>; and the remaining side you are touching is the <u>adjacent leg</u>.



Step 2: Write “SOH CAH TOA” on your paper:

SOH CAH TOA

Step 3: Ask yourself, “Which sides do I know?” In this example, we know that the <u>Adjacent leg</u> is 15”, and we know the <u>Hypotenuse</u> is 18”. To indicate that we know the <u>A</u>djacent leg, underline both <u>A</u>’s, and to indicate that we know the <u>Hypotenuse</u>, underline both <u>H</u>’s:

SO<u>H</u> C<u>AH</u> TO<u>A</u>

Step 4: Consider which of the three ratios has the most information: we have one piece of information for the sine and tangent (one underline each), yet we have two pieces of information for the cosine (two underlines). We are therefore going to use the cosine ratio formula:

$$\cos\theta = \frac{adjacent\ \ leg}{hypotenuse}$$

Step 5: Substitute the known information into the formula:

$$\cos y = \frac{adjacent\ \ leg}{hypotenuse} \Rightarrow \cos y = \frac{15''}{18''} \Rightarrow \cos y = \frac{15}{18} = \frac{5}{6} \quad (reduced)$$

Step 6: Determine the angle **y** that satisfies this equation.

Method 1: Table Lookup (approximate to the nearest degree)

We start with the equation

$$\cos y = \frac{15}{18} = \frac{5}{6}$$

First, we can approximate the fraction with a decimal:

$$\cos y = \frac{15}{18} = \frac{5}{6} = 0.8333$$

Next, we can examine a table of values from a chart and look for the closest "match" in the cosine column:

Angle	Sine	Cosine	Tangent
33°	0.54464	0.83867	0.64941
34°	0.55919	0.82904 ☜	0.67451
35°	0.57358	0.81915	0.70021

In which case, we pick an angle of 34°:

Angle	Sine	Cosine	Tangent
33°	0.54464	0.83867	0.64941
34° ☜	0.55919	**0.82904**	0.67451
35°	0.57358	0.81915	0.70021

So we conclude that y = 34° to the nearest degree.

Method 2: Inverse Function on a Calculator (more accurate)

We rewrite the equation using the inverse cosine as

$$y = \cos^{-1}\left(\tfrac{15}{18}\right) = \cos^{-1}\left(\tfrac{5}{6}\right)$$

which is pronounced "y is the inverse cosine of five-sixths."

What the inverse cosine function does is tell us what angle has a cosine of 5/6.

To enter this on a calculator, you typically type the SHIFT , INV or 2^{nd} key, and then the cos or $\cos^{-1}$ key (make sure your calculator is in degrees mode):

$$y = \cos^{-1}\left(\tfrac{5}{6}\right) = 33.5573°$$

You could also use an online inverse cosine calculator

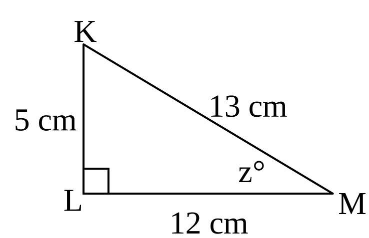
Step 7: Check for reasonableness: since the 15" side is almost as long as the hypotenuse, it makes sense that the angle would be less than 45°. This answer seems reasonable.

Note: it is also possible to directly check your answer in the equation:

$$\cos 33.5573° = \frac{15}{18}$$ ☺

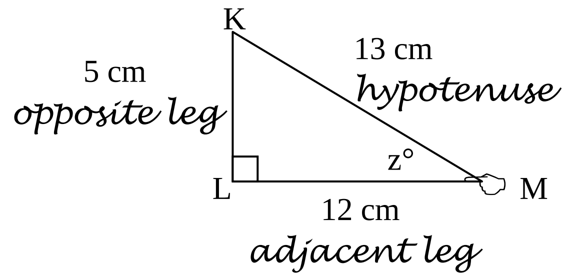
$$0.8333 = 0.8333$$

Example 3: Consider right ΔKLM pictured at right. We know all three sides, and our goal is to determine the degree measure of the unknown angle **z**.



www.MathWorksheetsGo.com

Step 1: Place your finger on the unknown angle z, and then label the three sides: the hypotenuse is always the longest side; the side you are not touching is the opposite leg; and the remaining side you are touching is the adjacent leg.



Step 2: Write "SOH CAH TOA" on your paper:

SOH CAH TOA

Step 3: Ask yourself, "Which sides do I know?" In this example, we know all three sides: The Adjacent leg is 12 cm, the Hypotenuse is 13 cm, and we know the Opposite leg is 5 cm. To indicate that we know the Adjacent leg, underline both A's, to indicate that we know the Opposite leg, underline both O's, and to indicate that we know the Hypotenuse, underline both H's:

S$\underline{OH}$ C$\underline{AH}$ T$\underline{OA}$

Step 4: Consider which of the three ratios has the most information: In this case, we could use any of them! For simplicity, let's favor the sine ratio in this example, but we'll carry the cosine and tangent ratios just to watch what happens:

$$\sin z = \frac{\text{opposite leg}}{\text{hypotenuse}}; \ \cos z = \frac{\text{adjacent leg}}{\text{hypotenuse}}; \tan z = \frac{\text{opposite leg}}{\text{adjacent leg}}$$

Step 5: Substitute the known information into the formula:

$$\sin z = \frac{5}{13}; \ \cos z = \frac{12}{13}; \tan z = \frac{5}{12}$$

Step 6: Determine the angle **z** that satisfies the equation.

Method 1: Table Lookup (approximate to the nearest degree)

We start with the equation

$$\sin z = \frac{5}{13};\ \cos z = \frac{12}{13};\ \tan z = \frac{5}{12}$$

Approximate the fractions with decimals:

$$\sin z = \frac{5}{13} = 0.3846;\ \cos z = \frac{12}{13} = 0.9231;$$

$$\tan z = \frac{5}{12} = 0.4167$$

Next, we can examine a

table of values from a chart

and look for the closest "match" in the sine column: (Notice how the cosine and tangent columns match as well!)

Angle	Sine	Cosine	Tangent
22°	0.37461	0.92718	0.40403
23°	0.39073 ☜	0.92050	0.42447
24°	0.40674	0.91355	0.44523

In which case, we pick an angle of 23°:

Angle	Sine	Cosine	Tangent
22°	0.37461	0.92718	0.40403
23° ☜	**0.39073**	0.92050	0.42447
24°	0.40674	0.91355	0.44523

So we conclude that z = 23° to the nearest degree.

Method 2: Inverse Function on a Calculator (more accurate)

We rewrite the equation using the inverse sine as

$$z = \sin^{-1}\left(\tfrac{5}{13}\right)$$

which is pronounced "z is the inverse sine of five-thirteenths."

What the inverse sine function does is tell us what angle has a sine of 5/13.

To enter this on a calculator, you typically type the SHIFT , INV or 2^{nd} key, and then the sin or $\sin^{-1}$ key (make sure your calculator is in degrees mode):

$$z = \sin^{-1}\left(\tfrac{5}{13}\right) = 22.6199°$$

You could also use an online inverse sine calculator

Note: Though this example favored the inverse sine, we could have used the inverse cosine or inverse tangent:

$$\cos^{-1}\left(\tfrac{12}{13}\right) = 22.6199°;\ \tan^{-1}\left(\tfrac{5}{12}\right) = 22.6199°,$$

both of which give identical answers.

Step 7: Check for reasonableness: since the 5 cm side is the shortest, it makes sense that the opposite angle is the smallest as well. ☺

Note: it is also possible to directly check your answer in the equations:

$$\sin 22.6199° = \frac{5}{13} \quad ☺$$
$$0.3846 = 0.3846$$

$$\cos 22.6199° = \frac{12}{13} \quad ☺$$
$$0.9231 = 0.9231$$

$$\tan 22.6199° = \frac{5}{12} \quad ☺$$
$$0.41671 = 0.4167$$

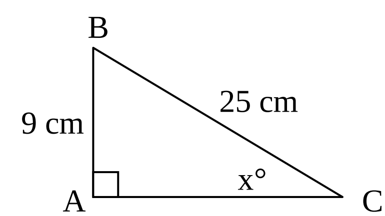
Part II) Practice Problems

1. Calculate the value of x to the nearest degree: $\sin x^\circ = 0.78801$

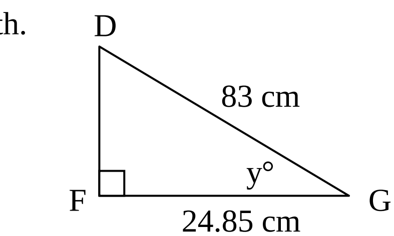
2. Calculate the value of y to the nearest tenth: $\cos y^\circ = \frac{24}{25}$

3. Calculate the value of z to the nearest hundredth: $\tan z^\circ = \frac{84.93}{34.627}$

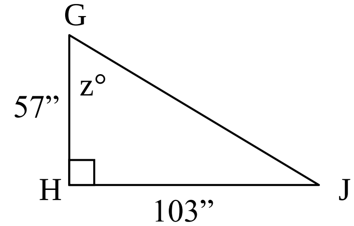
4. Determine the measure of angle x to the nearest tenth.



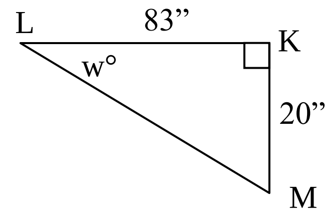
5. Determine the measure of angle y to the nearest hundredth.



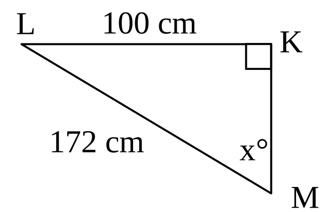
6. Determine the measure of angle z to the nearest degree.



7. Determine the measure of angle w to the nearest degree.



8. Error Analysis: Josh was asked to determine the measure of angle x to the nearest hundredth. His teacher marked it incorrect. His work is shown below. Find his error, and then correct it.



$$\sin x = \tfrac{100}{172}$$

rewrite :

$$x = \sin^{-1}(\tfrac{100}{172})$$

use a decimal approximation :

$$x = \sin^{-1}(0.58140)$$

if you raise it to the -1 *power, use a reciprocal* :

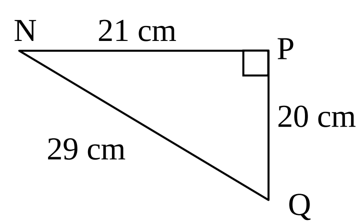
$$x = \frac{1}{\sin(0.58140)}$$

Simplify :

$$x = \frac{1}{0.01015}$$

$$x = 98.52°$$

9. For the triangle pictured, Marcy placed her finger on the vertex of angle N and concluded that $\cos N = \dfrac{21}{29}$. Likewise, Timmy placed his finger on the vertex of angle N and concluded that $\sin N = \dfrac{20}{29}$.



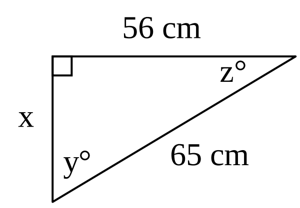
a) If you solve it beginning with Marcy’s equation, what answer will she get?

b) If you solve it Timmy’s way, what answer will he get?

c) Are these results reasonable? Explain.

Part III) Challenge Problems

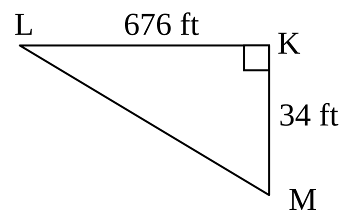
10. Use the Pythagorean Theorem, SOHCAHTOA, and the fact that the sum of the three interior angles of a triangle sum to 180° to determine all unknown sides and angles of the triangle pictured at right. Round all quantities, when necessary, to the nearest hundredth. (Note: you could also use an online Right Triangle Calculator.)



(drawing not to scale)

11. Error Analysis: Consider the right triangle pictured at right, which Annie and Lauren are both trying to solve. They both set it up using the equation $\tan M = \dfrac{676}{34}$

The steps of their work are shown below. Analyze their work to determine who, if anyone, is doing it correctly.



Annie's work	Lauren's work
$\tan L = \dfrac{676}{34}$ $L = \tan^{-1}\left(\dfrac{676}{34}\right)$ $L = 87.12°$	$\tan L = \dfrac{676}{34}$ $L = \tan\left(\dfrac{676}{34}\right)$ $L = 0.36°$

12. Consider the equation $\tan x° = \dfrac{74cm}{58cm}$

a) Sketch and label a right triangle that matches this equation.

b) Solve for x. Round to the nearest hundredth.

c) Determine the hypotenuse of your triangle. Round to the nearest hundredth.

d) Use the Pythagorean Theorem to confirm that this is, in fact, a right triangle.

13. Consider the following information: In ΔABC with right $\angle C$, the length of side AB is 42 cm, and the length of side AC is 20 cm.

a) Sketch and label a right triangle that matches this description.

b) Determine the measure of angle B to the nearest hundredth.

c) Determine the measure of the third angle.

14. For this problem, you will need to examine a chart of Sine, Cosine, and Tangent for each angle from 0° to 90°. You may use an online chart as well.

For each question, determine, to the nearest degree, the angle between 0° and 90° that most closely matches each description below:

a) The cosine of an angle is approximately twice as big as the sine of the same angle. (In other words, the number in the cosine column is very nearly twice as big as the number in the sine column.)

b) The tangent of an angle is approximately five times as big as the cosine of the same angle.

15. The following equations specify a specific right triangle: $\tan C = \frac{38}{40}$; $\cos B = \frac{38}{x}$; $\sin A = 1$

a) Make a labeled sketch of this triangle.

b) Determine the measure of angle B to the nearest tenth.

c) Determine the measure of angle C to the nearest tenth.

d) Determine the length of side x to the nearest hundredth.

16. A right triangle has an area of 54 cm^2 and one of its legs is 9 cm. Determine the measures of its two acute angles to the nearest degree.

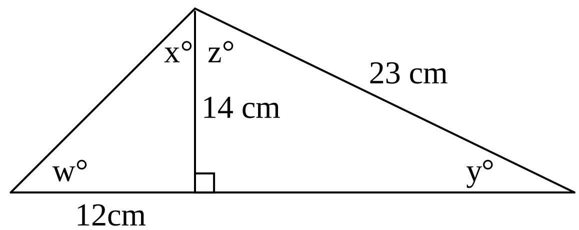
17. A 32-foot ladder is leaning against a tree. The base of the ladder rests 7 feet away from the foot of the tree. Assuming the tree is growing straight up:

a) Make a labeled sketch of the situation.

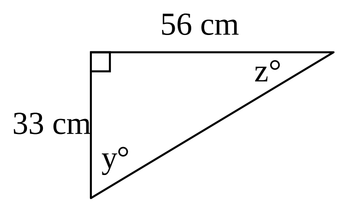
b) What acute angle does the ladder form with the ground?

c) How high up the tree does the ladder reach?

18. Determine the measures of angles w, x, y, and z. Round answers to the nearest hundredth:



19. Xavier, Yolanda and Zelda are examining the right triangle at right.



Yolanda observes that $\tan y = \frac{56}{33}$, so $y = \tan^{-1}(\frac{56}{33})$

and Zelda observes that $\tan z = \frac{33}{56}$, so $z = \tan^{-1}(\frac{33}{56})$

Xavier comments that $\frac{56}{33}$ and $\frac{33}{56}$ are reciprocals of each other.

a) To the nearest degree, what angle will Yolanda get for angle y?

b) To the nearest degree, what angle will Zelda get for angle z?

c) What is the sum of Yolanda’s and Zelda’s answers?

d) Explain why the answer you got in part (c) is reasonable. Think back to the good ol’ days of Geometry!

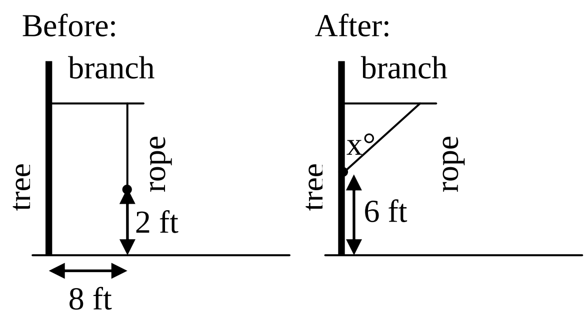
e) Xavier attempts to write a formula expressing this idea, but he gets stuck. Can you help him complete his formula?

$$\tan^{-1}\left(\tfrac{a}{b}\right) + \underline{\hspace{5cm}} = \underline{\hspace{5cm}}$$

f) Verify that your formula works when a = 20 and b = 7:

20. This problem will require you to assign variables to the unknown quantities and write equations to represent the given information. The Pythagorean Theorem will come in handy!

A rope swing (a rope with a large knot tied at the bottom, to sit on) is hanging straight down from a horizontal tree branch. In that position, the swing seat is two feet off the ground, and it hangs 8 feet from the tree. Ryan, who is 6 feet tall, pushes the swing seat toward the tree so that its edge is touching the tree, and Ryan realizes that he can fit directly underneath (it is as if he were wearing the seat as a hat!). Assume that the rope is still straight. A sketch of the situation is shown below:



a) How long is the rope?

b) How high up from the ground is the branch?

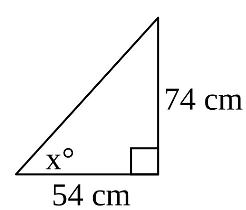
c) What angle x does the rope form with the tree branch?

Part IV) Answer Key

1. x = 52°
2. y = 16.3°
3. z = 67.82°
4. x = 21.1°
5. y = 72.58°
6. z = 61°
7. w = 14°
8. Josh incorrectly assumed that the expression "$\sin^{-1}(0.58140)$" means to take a reciprocal, so he wrote it as $\frac{1}{\sin(0.58140)}$, which is incorrect. Furthermore, he got an obtuse angle for what should be an acute angle. The correct value of $\sin^{-1}(0.58140) = 35.55°$, so x = 35.55°
9. a) 43.6°

   b) 43.6°

   c) These results are reasonable, because both of them are solving for the same angle in the same right triangle.
10. x = 33 cm; y = 59.49°; z = 30.51°
11. Annie is performing the steps correctly by using the inverse tangent. Lauren is using the tangent function, when the inverse tangent is called for.
12. a) One possible right triangle is shown:



   b) x = 53.88°

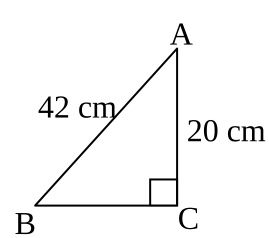
   c) 91.61 cm

   d)

$$\begin{aligned} 54^2 + 74^2 &= 91.61^2 \\ 2916 + 5476 &= 8392.3921 \\ 8392 &= 8392.3921 \end{aligned}$$

   This result is reasonable within roundoff error.

13. a) One possible right triangle is shown:



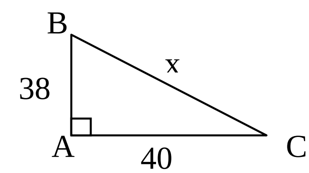
   b) the measure of $\angle B = 28.44°$

   c) the measure of $\angle A = 61.56°$

14. a) At approximately 27 degrees, the sine is 0.45, and the cosine is 0.89, which is almost twice as big.

b) At approximately 65 degrees, the cosine is 0.42, and the tangent is 2.14, which is approximately five times larger. $5(0.42) = 2.1$

15. a) Though the triangle may be oriented differently, the triangle must be equivalent to that drawn at right:



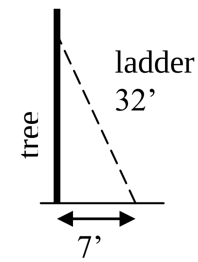
b) the measure of $\angle B = 46.5°$

c) the measure of $\angle C = 43.5°$

d) x = 55.17

16. 53° and 37°

17. a) A possible sketch of the ladder is shown at right:



b) 77°

c) 31.2 feet

18. w = 49.40°; x = 40.60°; y = 37.50°; z = 52.50°

19. a) y = 59°

b) z = 31°

c) 59° + 31° = 90°

d) Because the three interior angles of every triangle must sum to 90°, the two acute angles in a right triangle must sum to 90°.

e) $\tan^{-1}(\frac{a}{b}) + \tan^{-1}(\frac{b}{a}) = 90°$

f)

$$\begin{aligned} \tan^{-1}(\tfrac{20}{7}) + \tan^{-1}(\tfrac{7}{20}) &= 90° \\ 70.70995 + 19.29005 &= 90° \\ 90° &= 90° \end{aligned}$$ ☺

20. If we let y = the height of the branch, and z = the length of the rope, from the "before" picture, we may write: $y = z + 2$ since the branch must be two feet higher than the length of the rope.

From the "after" picture, we may use the Pythagorean Theorem to write $(y-6)^2 + 8^2 = z^2$.

Substituting $z + 2$ in place of y yields z = 10 feet, so y = 12 feet.

a) The rope is 10 feet long.

b) The branch is 12 feet above the ground.

c) $x = \sin^{-1}(\frac{8}{10}) = 53.13°$